WS 12 Review / Limits at Infinity

Quiz Review

1) a) $\lim_{x\to 2^-} f(x) = 2$

b) $\lim_{x\to 2^+} f(x) = 2$

c) $\lim_{x\to 2} f(x) = 2$

d) $f(2) = 1$

2) a) $\lim_{x\to 4^-} f(x) = 2$

b) $\lim_{x\to 4^+} f(x) = 4$

c) $\lim_{x\to 4} f(x)$ DNE

d) $f(4) = 4$

3) $\lim_{x\to 4}(x^2-3x+7) = 11$

4) $\lim_{x\to 1}\sqrt{x^3+7x+1} = 3$

5) $\lim_{x\to -3} \dfrac{x^2-9}{x^2-2x-15}$

$\lim_{x\to -3}(x^2-9) = 0 \quad \lim_{x\to -3}(x^2-2x-15)$

$\lim_{x\to -3} \dfrac{(x+3)(x-3)}{(x+3)(x-5)} = \lim_{x\to -3} \dfrac{x-3}{x-5}$

$= \boxed{\dfrac{6}{8}}$

6) $\lim_{x\to 0} \dfrac{2x^6+6x^3}{4x^5+3x^3}$

$\lim_{x\to 0}(2x^6+6x^3) = 0 \quad \lim_{x\to 0}(4x^5+3x^3) = 0$

$\lim_{x\to 0} \dfrac{x^3(2x^3+6)}{x^3(4x^5+3)} = 2$

7) $\lim_{x\to 3^-} \dfrac{|x-3|}{x-3} = -1$

8) $f(x) = \begin{cases} x^2\sin(\pi x), & x < 2 \\ x^2+cx-18, & x \geq 2 \end{cases}$

$\lim_{x\to 2^-}(x^2\sin(\pi x)) = \lim_{x\to 2^+}(x^2+cx-18)$

$4\sin 2\pi = 4+2c-18$

$0 = -14+2c$

$\boxed{c = 7}$

Limits at Infinity

1) $\lim_{x\to\infty}(-5x^3+7x^2+1) = -\infty$

2) $\lim_{x\to-\infty}(5x^3-7x^2+1) = -\infty$

3) $\lim_{x\to\infty}\left(8+\frac{9}{x^2}\right)$

$\lim_{x\to\infty} 8 + \lim_{x\to\infty}\frac{9}{x^2}$

$\boxed{8}$

4) $\lim_{x\to\infty}\frac{2+6x+9x^2}{x^2}$

$\lim_{x\to\infty}\frac{9x^2}{x^2} = \boxed{9}$

5) $\lim_{x\to\infty}\frac{9x+8}{9x+7}$

$\lim_{x\to\infty}\frac{9x}{9x} = \boxed{1}$

6) $\lim_{x\to-\infty}\frac{9x+8}{9x+7}$

$\lim_{x\to-\infty}\frac{9x}{9x} = \boxed{1}$

7) $\lim_{x\to\infty}\frac{\sqrt{2x^2-4}}{x+3}$

$\lim_{x\to\infty}\frac{\sqrt{2x^2}}{x}$

$\lim_{x\to\infty}\frac{x\sqrt{2}}{x} = \boxed{\sqrt{2}}$

8) $\lim_{x\to-\infty}\frac{\sqrt{x^2-4}}{x+3}$

$\lim_{x\to-\infty}\frac{\sqrt{x^2}}{x}$

$\lim_{x\to-\infty}\frac{|x|}{x} = \boxed{-1}$

9) $\lim_{x\to\infty}\frac{x^3}{e^{3x}} = \boxed{0}$

10) $\lim_{x\to-\infty}\frac{x^3}{e^{3x}}$

$\frac{\lim_{x\to-\infty} x^3}{\lim_{x\to-\infty} e^{3x}} \to \frac{-\infty}{0} = \boxed{-\infty}$

11) $\lim_{x\to\infty}\frac{\sin 3x}{15x} = \boxed{0}$

12) $\lim_{x\to-\infty}\frac{7-6x+\sin 2x}{6x+\cos 2x}$

$\lim_{x\to-\infty}\frac{-6x}{6x} = \boxed{-1}$

13) I, III

14) a) $y = \frac{20x^2-x}{1+4x^2}$

$\lim_{x\to\infty}\frac{20x^2-x}{1+4x^2} = 5$

H.A. $y=5$

b) $f(x) = \frac{(3x+8)(5-4x)}{(2x+1)^2}$

$\lim_{x\to\infty} f(x) \Rightarrow \lim_{x\to\infty}\frac{-12x^2}{4x^2} = -3$

H.A. $y=-3$

c) $\lim_{x\to\infty}\frac{x}{|x|} = 1$

$\lim_{x\to-\infty}\frac{x}{|x|} = -1$

H.A.
$y=1$
$y=-1$